Chapter 1

Exercise 1.1

Q. 1. (i) $2(3)(-4) = -24$

(ii) $(-4 + 7)^3$

$= 3^3$

$= 27$

(iii) $\dfrac{-4 + 3}{2(7)}$

$= -\dfrac{1}{14}$

(iv) $3(-4)(7) - (-4)^2$

$= -84 - 16$

$= -100$

(v) $\sqrt{\dfrac{q^2 + rp + r + 4}{-\frac{q}{p}}}$

$\sqrt{\dfrac{48}{\frac{4}{3}}} = \sqrt{36} = 6$

Q. 2. (i) $2x - 3y + 4x - 2y + 3y + 4x$

$= 6x + 4x - 3y - 2y + 3y$

$= 10x - 2y$

(ii) $2xy + 3xy - 7xy$

$= 5xy - 7xy$

$= -2xy$

(iii) $4pq + 4qr + 5pq - 7qr - 8qp$

$= 4pq + 5pq - 8pq + 4qr - 7qr$

Note $-8pq = -8qp$

$= pq - 3qr$

(iv) $p^2 + p^2 + 2p^3$

$= 2p^2 + 2p^3$

(v) $xy^2 - 2yx^2 + y^2x$

$= xy^2 + xy^2 - 2x^2y$

$= 2xy^2 - 2x^2y$

(vi) $5nm^2 + 2mn^2 - 2m^2n - 3mn^2$

$= 5m^2n - 2m^2n + 2mn^2 - 3mn^2$

$= 3m^2n - mn^2$

Q. 3. (i) $3x^2 + 12x + 15x - 10$

$= 3x^2 + 27x - 10$

(ii) $3a^2 - 3b - a + 3b$

$= 3a^2 - a$

(iii) $36x^3 + 24x^2 + 12x + 10x^2 - 20x$

$= 36x^3 + 34x^2 - 8x$

(iv) $-2y^2 + 3xy - xy^2 - 3xy$

$= -2y^2 - xy^2$

(v) $b^3 + 4b^2 + bc - 4a^2c - 4bc$

$= b^3 + 4b^2 - 4a^2c - 3bc$

Q. 4. (i) $x^2 - 3x + 2x - 6$

$= x^2 - x - 6$

(a) Degree: 2 (b) Constant: −6 (c) 3 terms (d) −1

(ii) $6x^2 + 8x - 15x - 20$

$= 6x^2 - 7x - 20$

(a) Degree: 2 (b) Constant: −20 (c) 3 terms (d) −7

(iii) $(-5x^3 + 3x)(2x^2 - 3)$

$= -10x^5 + 15x^3 + 6x^3 - 9x$

$= -10x^5 + 21x^3 - 9x$

(a) Degree: 5 (b) Constant: none (c) 3 terms (d) −9

(iv) $2x^5 - 8x^4 + 3x^3 - 8x^2 + 32x - 12$

(a) Degree: 5 (b) Constant: −12 (c) 6 terms (d) 32

(v) $44x^5 - 12x^4 + 4x^3 - 12x^2 + 4x - 77x^4 + 21x^3 - 7x^2 + 21x - 7$

$= 44x^5 - 89x^4 + 25x^3 - 19x^2 + 25x - 7$

(a) Degree: 5 (b) Constant: −7 (c) 6 terms (d) 25

Q. 5. (i) $35x^2 + 12x + 1$

(ii) $p^2 - q^2$

(iii) $(15s - 5t)(2t - 1)$

$= 30st - 15s - 10t^2 + 5t$

(iv) $p^2 - 2pq + q^2$

(v) $2x(16x^2 + 24x + 9)$

$= 32x^3 + 48x^2 + 18x$

(vi) $x^3 - x^2y - xy^2 + y^3$

(vii) $y^3 - 3y^2(3) + 3(y)(9) - 3^3$

$= y^3 - 9y^2 + 27y - 27$

(viii) $(2a)^3 + 3(2a)^2(5b) + 3(2a)(5b)^2 + (5b)^3$

$= 8a^3 + 60a^2b + 150ab^2 + 125b^3$

(ix) $(9x)^3 - 3(9x)^2(2y) + 3(9x)(2y)^2 - (2y)^3$

$= 729x^3 - 486yx^2 + 108xy^2 - 8y^3$

Q. 6. (i) $(a + 1)^4$

$= 1a^4(1^0) + 4a^3(1^1) + 6a^2(1^2) + 4a(1^3) + 1a^0(1^4)$

$= a^4 + 4a^3 + 6a^2 + 4a + 1$

(ii) $(b - 3)^3$

$= (b + (-3))^3$

$= 1b^3 + 3b^2(-3)^1 + 3(b^1)(-3)^2 + 1(b^0)(-3)^3$

$= b^3 - 9b^2 + 27b - 27$

(iii) $(x + y)^5$

$= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

(iv) $(2a + 3b)^3$

$= 1(2a)^3 + 3(2a)^2(3b) + 3(2a)(3b)^2 + 1(2a)^0(3b)^3$

$= 8a^3 + 36a^2b + 54ab^2 + 27b^3$

(v) $(3y + (-4x))^4$

$= 1(3y)^4 + 4(3y)^3(-4x) + 6(3y)^2(-4x)^2 + 4(3y)^1(-4x)^3 + 1(-4x)^4$

$= 81y^4 - 432xy^3 + 864x^2y^2 - 768x^3y + 256x^4$

(vi) $1(3x)^5 + 5(3x)^4(-2y) + 10(3x)^3(-2y)^2 + 10(3x)^2(-2y)^3 + 5(3x)^1(-2y)^4 + 1(-2y)^5$

$= 243x^5 - 810x^4y + 1{,}080x^3y^2 - 720x^2y^3 + 240xy^4 - 32y^5$

Q. 7. (i) Area $= (x + 2)^2$

$= x^2 + 4x + 4$

Perimeter $= 4(x + 2)$

$= 4x + 8$

(ii) Area $= (x + y)(2x - 3y)$

$= 2x^2 - 3xy + 2xy - 3y^2$

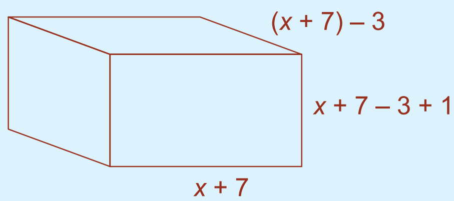
$= 2x^2 - xy - 3y^2$

Perimeter $= 2(x + y) + 2(2x - 3y)$

$= 2x + 2y + 4x - 6y$

$= 6x - 4y$

Q. 8.



Volume: $(x + 7)(x + 4)(x + 5)$

$= (x^2 + 11x + 28)(x + 5)$

$= x^3 + 5x^2 + 11x^2 + 55x + 28x + 140$

Answer: $x^3 + 16x^2 + 83x + 140$

Surface Area:

$2(x + 7)(x + 5)$ $2(x^2 + 12x + 35)$

$2x^2 + 24x + 70$

$+ 2(x + 4)(x + 5)$ + $2x^2 + 18x + 40$

$+ \underline{1\ (x + 7)(x + 4)}$ + $\underline{x^2 + 11x + 28}$

Answer: $= \underline{5x^2 + 53x + 138}$

Q. 9. x = correct answer

y = total attempts

$8x - 3(y - x) + (20 - y)$

$= 8x - 3y + 3x + 20 - y$

$= 11x - 4y + 20$

Q. 10. Mark $= y$

5 years time ⇒ Aoife $2(y + 5)$

Daniel ⇒ 2 yrs younger than Aoife now

Aoife now $= 2(y + 5) - 5$

Daniel 5 yrs $= [2(y + 5) - 5] - 2$

Daniel now $= (2(y + 5) - 5) - 2 - 5$

sum of ages $= y + 2\ (y + 5) -5 + 2(y + 5) -5 - 7$

$= 5y + 10 - 5 + 10 - 12$

$= 5y + 3$

Q. 11. (i) $(a + b)(x^2 + 10x + 25)$

$= ax^2 + 10ax + 25a + bx^2 + 10bx + 25b$

(ii) $3p(p^2 - q^2)$

$= 3p^3 - 3pq^2$

(iii) $(z^2 + 2xz + x^2)(z^2 - 2xz + x^2)$

$= z^4 - 2xz^3 + x^2z^2 + 2xz^3 - 4x^2z^2 + 2x^3z + x^2z^2 - 2x^3z + x^4$

$= z^4 - 2x^2z^2 + x^4$

OR

$(z + x)(z + x)(z - x)(z - x)$

$= (z + x)(z - x)(z + x)(z - x)$

$= (z^2 - x^2)(z^2 - x^2)$

$= (z^2 - x^2)^2$

$= z^4 - 2x^2z^2 + x^4$

(iv) $(3a^2 + 9ab - a - 3b)(2a + b)$

$= 6a^3 + 3a^2b + 18a^2b + 9ab^2 - 2a^2 - ab - 6ab - 3b^2$

$= 6a^3 + 21a^2b + 9ab^2 - 2a^2 - 7ab - 3b^2$

(v) $5(x^2 + 2xy + y^2)(x + 4y)$

$= 5(x^3 + 4x^2y + 2x^2y + 8xy^2 + xy^2 + 4y^3)$

$= 5x^3 + 20x^2y + 10x^2y + 40xy^2 + 5xy^2 + 20y^3$

$= 5x^3 + 30x^2y + 45xy^2 + 20y^3$

Q. 12. $14{,}580x^3y^3$

$(ax + ay)^n \Rightarrow n = 6$

1 6 15 [20] 15 6 1

$20(ax)^3(ay)^3 = 14{,}580x^3y^3$

$20a^6\cancel{x^3}\cancel{y^3} = 14{,}580\cancel{x^3}\cancel{y^3}$

$20a^6 = 14{,}580$

$a^6 = 729$

$a = \pm 3$

But $a \in N$: $\therefore\ a = 3$

Exercise 1.2

Q. 1. $4ab^2(1 - 3b)$

Q. 2. $(7x + 2)(x + 1)$

Q. 3. $(3y - 7)(y + 1)$

Q. 4. $(5x + 2)(x + 2)$

Q. 5. $(x + 3)(x - 6)$

Q. 6. $(3x - 2)(x + 4)$

Q. 7. $(2y - 7)(y + 9)$

Q. 8. $(7x - 19)(x + 3)$

Q. 9. $(5a)^2 - 1$

$= (5a - 1)(5a + 1)$

Q. 10. $2x^2 - 9x + 4$

$= (2x - 1)(x - 4)$

Q. 11. $12xy - 21x - 8y + 14$

$= (3x(4y - 7) - 2(4y - 7))$

$= (3x - 2)(4y - 7)$

Q. 12. $(5x + 12)(x + 8)$

Q. 13. $(8a)^2 - (9b)^2$

$= (8a - 9b)(8a + 9b)$

Q. 14. $(2x + 1)(x - 12)$

Q. 15. $(4x + 3)(x + 1)$

Q. 16. Rearrange as $6a^2 + 4ac - 15ab - 10bc$

$= 2a(3a + 2c) - 5b(3a + 2c)$

$= (2a - 5b)(3a + 2c)$

Q. 17. $(10y + 17)(y + 1)$

Q. 18. $(3x + 2)(3x - 1)$

Q. 19. $(3x - 2)(3x - 5)$

Q. 20. $(4y + 19)(y + 1)$

Q. 21. Rearrange as $x(y - z) - 1(y - z)$

$= (x - 1)(y - z)$

Q. 22. $6 \times 45 = 270$

27 and 10

$6x^2 + 27x + 10x + 45$

$= 3x(2x + 9) + 5(2x + 9)$

$= (3x + 5)(2x + 9)$

Q. 23. $(6p - 10q)(6p + 10q)$

$= 4(3p - 5q)(3p + 5q)$

Q. 24. $(4x - 7)(3x + 8)$

Q. 25. $8x^2 - 10x - 12x + 15$

$= 2x(4x - 5) - 3(4x - 5)$

$= (2x - 3)(4x - 5)$

Q. 26. $10x^2 - 4x - 6$

$= 10x^2 - 10x + 6x - 6$

$= 10x(x - 1) + 6(x - 1)$

$= (10x + 6)(x - 1)$

$= 2(5x + 3)(x - 1)$

Q. 27. $12x^2 - 24x + 6x - 12$

$= 12x(x - 2) + 6(x - 2)$

$= (12x + 6)(x - 2)$

$= 6(2x + 1)(x - 2)$

Q. 28. $12y(4y^2 + 4y + 1)$

$= 12y(4y^2 + 2y + 2y + 1)$

$= 12y(2y(2y + 1) + 1(2y + 1))$

$= 12y(2y + 1)(2y + 1)$

Q. 29. $(4x - 10)(4x + 10)$

$= 4(2x - 5)(2x + 5)$

Q. 30. $2x(x^2 - 4)$

$= 2x(x - 2)(x + 2)$

Q. 31. $p(6q^2 + 11q + 4)$

$= p(3q + 4)(2q + 1)$

Q. 32. $x^2(x^2 - 36)$

$= x^2(x - 6)(x + 6)$

Rule $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Q. 33. $(x - 3)(x^2 + 3x + 9)$

Rule $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Q. 34. $(p + 2)(p^2 - 2p + 4)$

Q. 35. $(x - y)(x^2 + xy + y^2)$

Q. 36. $(4a)^3 - 1^3$

$= (4a - 1)(16a^2 + 4a + 1)$

Q. 37. $(2a)^3 + (3b)^3$

$= (2a + 3b)(4a^2 - 6ab + 9b^2)$

Q. 38. $(5p)^3 + (8q)^3$

$= (5p + 8q)(25p^2 - 40pq + 64q^2)$

Q. 39. $(10x)^3 - 9^3$

$= (10x - 9)(100x^2 + 90x + 81)$

Q. 40. $(7c)^3 + d^3$

$= (7c + d)(49c^2 - 7cd + d^2)$

Q. 41. $3(x - 6)(x^2 + 6x + 36)$

Q. 42. $16(8 + x^3)$

$= 16(2^3 + x^3)$

$= 16(2 + x)(4 - 2x + x^2)$

Q. 43. $54a(a^3 + 8b^3)$

$= 54a(a^3 + (2b)^3)$

$= 54a(a + 2b)(a^2 - 2ab + 4b^2)$

Q. 44. $(x + 2 + x + 3)(x + 2 - x - 3)$

$= (2x + 5)(-1)$

$= -2x - 5$

OR

$\cancel{x^2} + 4x + 4 - \cancel{x^2} - 6x - 9$

$= -2x - 5$

Q. 45. $(x + p)(x + p)$

Q. 46. $(ac - b)(ac + b)$

Q. 47. $(x^2 - 5)(x^2 + 5)$

Q. 48. $(ab - 1)(ab - 1)$

Q. 49. $(x^2 + y^2)(x - y)(x + y)$

Q. 50. $(x - y + 3)(x - y - 3)$

Q. 51. $(4x - 3y)(2x - 3y)$

Q. 52. $a(a^3 + 1)$

$= a(a + 1)(a^2 - a + 1)$

Q. 53. $[a - (b + c)][a + (b + c)]$

Q. 54. $ab^2(b^3 - 1)$

$= ab^2(b - 1)(b^2 + b + 1)$

Q. 55. $(x^2 + 2)(x - 3)(x + 3)$

Exercise 1.3

Q. 1. $\dfrac{4x - 1 + 4x - 10}{4}$

$= \dfrac{8x - 11}{4}$

Q. 2. $\dfrac{3x - 7 - 15x + 9}{12}$

$= -\dfrac{12x + 2}{12}$

$= -\dfrac{6x + 1}{6}$

Q. 3. $\dfrac{7 + 8x - 10}{4x - 5}$

$= \dfrac{-3 + 8x}{4x - 5}$

Q. 4. $\dfrac{7 - 10}{35x}$

$= -\dfrac{3}{35x}$

Q. 5. $\dfrac{8 + x^2 + x}{x + 1}$

Q. 6. $\dfrac{3x - x - 5}{x(x + 5)}$

$= \dfrac{2x - 5}{x^2 + 5x}$

Q. 7. $\dfrac{5(3x - 1) + 2(x - 2)}{(x - 2)(3x - 1)}$

$= \dfrac{15x - 5 + 2x - 4}{(x - 2)(3x - 1)}$

$= \dfrac{17x - 9}{3x^2 - 7x + 2}$

Q. 8. $\dfrac{2x - 1 + 9x + 15}{(3x + 5)(2x - 1)}$

$= \dfrac{11x + 14}{6x^2 + 7x - 5}$

Q. 9. $\dfrac{\cancel{ab}(b^2)}{\cancel{ab}(a)}$

$= \dfrac{b^2}{a}$

Q. 10. $\dfrac{\cancel{x + y}}{(\cancel{x + y})(x - y)}$

$= \dfrac{1}{x - y}$

Q. 11. $\dfrac{b(\cancel{a + b})}{(a - b)(\cancel{a + b})}$

$= \dfrac{b}{a - b}$

Q. 12. $\dfrac{p - q}{q - p}$

$= \dfrac{-1(\cancel{-p + q})}{(\cancel{q - p})}$

$= -1$

Q. 13. $\dfrac{x(x-2)}{x-2}$

$= x$

Q. 14. $\dfrac{5x(3x+1)}{(5x+5)(3x+1)}$

$= \dfrac{5x}{5x+5} = \dfrac{5(x)}{5(x+1)} = \dfrac{x}{x+1}$

Q. 15. $\dfrac{(x-2)(x^2+2x+4)}{(x^2+2x+4)}$

$= x - 2$

Q. 16. $\dfrac{2x^4 - 250x}{4x^2 - 20x}$

$= \dfrac{x^2 + 5x + 25}{2}$

Q. 17. $\dfrac{(x+y)(x^2 - xy + y^2)}{(x+y)(x-y)}$

$= \dfrac{x^2 - xy + y^2}{x - y}$

Q. 18. $\dfrac{(x^2 - y^2)(x^2 + y^2)}{(x^2 - y^2)}$

$= x^2 + y^2$

Q. 19. $\dfrac{(p-q)(p^2 - 2pq + q^2)}{(p-q)(p+q)}$

$= \dfrac{p^2 - 2pq + q^2}{p + q}$

Q. 20. $\dfrac{x^2 - 8x + 16}{x^2 - 16} + \dfrac{16x}{x^2 - 16}$

$= \dfrac{x^2 + 8x + 16}{x^2 - 16} = \dfrac{(x+4)(x+4)}{(x-4)(x+4)}$

$= \dfrac{x+4}{x-4}$

Q. 21. $\dfrac{2x}{x-1} + \dfrac{x}{x-1}$

$= \dfrac{3x}{x-1}$

OR

$\dfrac{2x(1-x) - x(x-1)}{(x-1)(1-x)}$

$= \dfrac{2x - 2x^2 - x^2 + x}{x - x^2 - 1 + x}$

$= \dfrac{-3x^2 + 3x}{-x^2 + 2x - 1}$

$= \dfrac{3x^2 - 3x}{x^2 - 2x + 1}$

$= \dfrac{3x(x-1)}{(x-1)(x-1)}$

$= \dfrac{3x}{x-1}$

Q. 22. $\dfrac{x+4}{(x-4)(x+4)} + \dfrac{x-5}{(x-5)(x+5)}$

$= \dfrac{1}{x-4} + \dfrac{1}{x+5}$

$= \dfrac{x+5+x-4}{(x-4)(x+5)}$

$= \dfrac{2x+1}{x^2 + x - 20}$

Q. 23. $\dfrac{x - 3 - 3(x+2) + 4}{(x+2)(x-3)}$

$= \dfrac{x - 3 - 3x - 6 + 4}{(x+2)(x-3)}$

$= \dfrac{-2x - 5}{(x+2)(x-3)}$

Q. 24. $\dfrac{1}{x^2 + 3x + 2} + \dfrac{4x + 4 - 3x - 6}{x^2 + 3x + 2}$

$= \dfrac{1 + x - 2}{x^2 + 3x + 2}$

$= \dfrac{x - 1}{x^2 + 3x + 2}$

Q. 25. $\dfrac{3}{a-1} + \dfrac{a-1}{a+1} - \dfrac{a+1}{a^2-1}$

$= \dfrac{3a + 3 + (a-1)^2}{a^2 - 1} - \dfrac{a+1}{a^2-1}$

$= \dfrac{2a + 2 + a^2 - 2a + 1}{a^2 - 1}$

$= \dfrac{a^2 + 3}{a^2 - 1}$

Q. 26. $\dfrac{a^2 - ab + a^2 + ab}{a^2 - b^2}$

$= \dfrac{2a^2}{a^2 - b^2}$

Q. 27. $\dfrac{(a+2)(a+3)+(a-3)(a-2)}{(a-2)(a+3)}$

$= \dfrac{a^2+\cancel{5a}+6+a^2-\cancel{5a}+6}{(a-2)(a+3)}$

$= \dfrac{2a^2+12}{(a-2)(a+3)}$

Q. 28. $\dfrac{n(n+2)+(n+2)(n+1)+n(n+1)}{(n+2)(n+1)(n)}$

$= \dfrac{n^2+2n+n^2+3n+2+n^2+n}{n(n+1)(n+2)}$

$= \dfrac{3n^2+6n+2}{n(n+1)(n+2)}$

Q. 29. $\dfrac{a+b+a-b}{a^2-b^2}$

$= \dfrac{2a}{a^2-b^2}$

OR

$= \dfrac{\cancel{a+b}}{(a-b)\cancel{(a+b)}} - \dfrac{a-b}{(b-a)(b+a)}$

$= \dfrac{(b-a)(b+a)-(a-b)^2}{(a-b)(b-a)(b+a)}$

$= \dfrac{\cancel{b^2}-a^2-a^2+2ab-\cancel{b^2}}{(a-b)(b-a)(b+a)}$

$= \dfrac{-2a\cancel{(a-b)}}{\cancel{(a-b)}(b-a)(b+a)}$

$= \dfrac{-2a}{b^2-a^2}$

$= \dfrac{2a}{a^2-b^2}$

Q. 30. $\dfrac{a-b}{a^2-2ab+b^2} - \dfrac{a+b}{a^2+2ab+b^2}$

$= \dfrac{^1\cancel{a-b}}{_1\cancel{(a-b)}(a-b)} - \dfrac{\cancel{a+b}^1}{_1\cancel{(a+b)}\,(a+b)}$

$= \dfrac{1}{a-b} - \dfrac{1}{a+b}$

$= \dfrac{1(a+b)-1(a-b)}{(a-b)(a+b)} = \dfrac{2b}{(a-b)(a+b)}$

Q. 31. (i) $\dfrac{5}{x^2-1} + \dfrac{1}{1-x^2} = \dfrac{5-1}{x^2-1} = \dfrac{4}{x^2-1}$

(ii) $\dfrac{3x}{x^2+3x-18} + \dfrac{18}{x^2+3x-18}$

$= \dfrac{3x+18}{(x+6)(x-3)}$

$= \dfrac{3\cancel{(x+6)}}{\cancel{(x+6)}(x-3)}$

$= \dfrac{3}{x-3}$

Exercise 1.4

Q. 1. $\dfrac{{}^{5}\cancel{10b^2}}{\cancel{5a^2}} \times \dfrac{{}^{5a}\cancel{25a^3}}{\cancel{2b}}$

$= \dfrac{25ab}{1}$

$= 25ab$

Q. 2. $\dfrac{\cancel{4x}}{{}_1\cancel{2y}} \times \dfrac{{}^2\cancel{4y^2}}{{}_2\cancel{8x^3}^2} = \dfrac{2y}{2x^2} = \dfrac{y}{x^2}$

Q. 3. $\dfrac{\cancel{2}}{\cancel{2x-1}} \times \dfrac{\cancel{(2x-1)}(2x+1)}{\cancel{4}\,2}$

$= \dfrac{2x+1}{2}$

Q. 4. $\dfrac{x^2+x-2}{x^2+2x-3} \times \dfrac{2x+6}{4x-4}$

$= \dfrac{\cancel{(x-1)}(x+2)}{(x-1)\cancel{(x+3)}} \times \dfrac{\cancel{(2)}\cancel{(x+3)}}{\cancel{(4)}_2\cancel{(x-1)}}$

$= \dfrac{x+2}{2x-2}$

Q. 5. $\dfrac{4x-4}{x} \times \dfrac{x^3}{x^2-1}$

$= \dfrac{4\cancel{(x-1)}(x^2)}{\cancel{(x-1)}(x+1)}$

$= \dfrac{4x^2}{x+1}$

Q. 6. $\dfrac{\cancel{(y-8)}(y+8)\cancel{(2}y)\cancel{(y-4)}}{\cancel{(y-4)}(y+4)\cancel{(2)}\cancel{(y-8)}}$

$= \dfrac{y^2+8y}{y+4}$

Q. 7. $\dfrac{(2x+1)\cancel{(x-1)}\cancel{(2x-1)}(2x+1)}{\cancel{(2x-1)}(x+1)\cancel{(x-1)}(x+1)}$

$= \dfrac{(2x+1)^2}{(x+1)^2}$

Q. 8. $\dfrac{8x^2-34x-9}{4x+1} \times \dfrac{3x}{4x^2-81}$

$= \dfrac{(8x^2-36x+2x-9)(3x)}{(4x+1)(2x-9)(2x+9)}$

$= \dfrac{[2x(4x+1)-9(4x+1)](3x)}{(4x+1)(2x-9)(2x+9)}$

$= \dfrac{\cancel{(2x-9)}\cancel{(4x+1)}(3x)}{\cancel{(4x+1)}\cancel{(2x-9)}(2x+9)}$

$= \dfrac{3x}{2x+9}$

Q. 9. $\frac{6x^2 - 20x + 16}{4x^2 - 16x + 16} \times \frac{2x^2 + 2x - 12}{9x^2 - 16}$

$= \frac{{}^{1}\cancel{2}(\cancel{3x-4})(\cancel{x-2})}{{}_{1}\cancel{4}_{\cancel{2}}(\cancel{x-2})(\cancel{x-2})} \times \frac{\cancel{2}(x+3)(\cancel{x-2})}{(\cancel{3x-4})(3x+4)}$

$= \frac{x+3}{3x+4}$

Q. 10. $\frac{\cancel{4x+3}}{(\cancel{x-7})(x+7)} \times \frac{\cancel{x-7}}{(4x-3)(\cancel{4x+3})}$

$= \frac{1}{(x+7)(4x-3)}$

Q. 11. $\frac{x-y}{5}$

Q. 12. $\frac{x-5}{\cancel{x+5}} \times \frac{(x-5)(\cancel{x+5})}{1}$

$= (x-5)^2$

Q. 13. $\frac{\frac{x+x+1}{x+1}}{\frac{3}{x+1}}$

$= \frac{2x+1}{3}$

Q. 14. $\frac{1 - \frac{1}{x}}{2 - \frac{2}{x^2}}$

$= \frac{\frac{x-1}{x}}{\frac{2x^2-2}{x^2}} = \frac{\cancel{x-1}}{\cancel{x}_1} \times \frac{x^{\cancel{2}}}{2(\cancel{x-1})(x+1)}$

$= \frac{x}{2(x+1)}$

Q. 15. $\frac{x}{(x-3)(\cancel{x-3})} \times \frac{(3-x)(\cancel{x-3})}{5}$

$= -\frac{x(\cancel{x-3})}{5(\cancel{x-3})}$

$= -\frac{x}{5}$

Q. 16. $\frac{ab-1}{b-1}$

Q. 17. $\frac{p+q}{\frac{p+q}{pq}}$

$= (\cancel{p+q}) \times \frac{pq}{\cancel{p+q}}$

$= pq$

Q. 18. $\frac{\cancel{x+3}}{y-x} \times \frac{(x-y)(x+y)}{x(\cancel{x+3})}$

$= \frac{(x-y)(x+y)}{x(y-x)}$

$= \frac{(\cancel{x-y})(x+y)}{-x(\cancel{x-y})}$

$= -\frac{x+y}{x}$

Q. 19. $\frac{(x-y)(\cancel{x+y})}{x^{\cancel{2}}} \times \frac{\cancel{x}}{(x+y)(\cancel{x+y})}$

$= \frac{x-y}{x(x+y)}$

Q. 20. $\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$

Denominator $\frac{1}{R_1} + \frac{1}{R_2}$

$= \frac{R_2 + R_1}{R_1R_2}$

$R_T = \frac{1}{\frac{R_2 + R_1}{R_1R_2}} = \frac{1}{1} \times \frac{R_1R_2}{R_2 + R_1}$

$= \frac{R_1R_2}{R_1 + R_2}$

Q. 21. $u = x - \frac{1}{x}$

$u^2 = \left(x - \frac{1}{x}\right)^2$

$= x^2 - \frac{2x}{x} + \frac{1}{x^2}$

$= x^2 - 2 + \frac{1}{x^2}$

Let $u^2(u^2 + 4) = v^2$

$v = x^2 - \frac{1}{x^2}$

$v^2 = x^4 - 2 + \frac{1}{x^4}$

LHS

$x^2\left(x^2 - 2 + \frac{1}{x^2}\right) - 2x^2 + 4 - \frac{2}{x^2} + 1 - \frac{2}{x^2} + \frac{1}{x^4} + 4x^2 - 8 + \frac{4}{x^2}$

$= x^4 - \cancel{2x^2} + 1 - \cancel{2x^2} + 4 - \cancel{\frac{2}{x^2}} + 1 - \cancel{\frac{2}{x^2}} + \frac{1}{x^4} + \cancel{4x^2} - 8 + \cancel{\frac{4}{x^2}}$

$= x^4 - 2 + \frac{1}{x^4} = v^2$

$\therefore$ LHS = RHS

Q. 22. $\frac{a^3 - b^3}{a^3 + ab^2} \times \frac{a^2b + b^3}{a^3 - ab^2}$

$= \frac{(a - b)(a^2 + ab + b^2) \times (b)\cancel{(a^2 + b^2)}}{a\cancel{(a^2 + b^2)}(a)(a^2 - b^2)}$

$= \frac{\cancel{(a - b)}(b)(a^2 + ab + b^2)}{a^2\cancel{(a - b)}(a + b)}$

$= \frac{b(a^2 + ab + b^2)}{a^2(a + b)}$

Exercise 1.5

Q. 1. (i) $(x + y)^5 = \binom{5}{0}x^5 + \binom{5}{1}x^4y + \binom{5}{2}x^3y^2 + \binom{5}{3}x^2y^3 + \binom{5}{4}xy^4 + \binom{5}{5}y^5$

$= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

(ii) $(x + y)^6 = \binom{6}{0}x^6 + \binom{6}{1}x^5y + \binom{6}{2}x^4y^2 + \binom{6}{3}x^3y^3 + \binom{6}{4}x^2y^4 + \binom{6}{5}xy^5 + \binom{6}{6}y^6$

$= x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$

(iii) $(x + y)^7 = \binom{7}{0}x^7 + \binom{7}{1}x^6y + \binom{7}{2}x^5y^2 + \binom{7}{3}x^4y^3 + \binom{7}{4}x^3y^4 + \binom{7}{5}x^2y^5 + \binom{7}{6}xy^6 + \binom{7}{7}y^7$

$= x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$

(iv) $(a + b)^8 = \binom{8}{0}a^8 + \binom{8}{1}a^7b + \binom{8}{2}a^6b^2 + \binom{8}{3}a^5b^3 + \binom{8}{4}a^4b^4 + \binom{8}{5}a^3b^5 + \binom{8}{6}a^2b^6 + \binom{8}{7}ab^7 + \binom{8}{8}b^8$

$= a^8 + 8a^7b + 28a^6b^2 + 56a^5b^3 + 70a^4b^4 + 56a^3b^5 + 28a^2b^6 + 8ab^7 + b^8$

(v) $(1 + 2x)^3 = \binom{3}{0}(1)^3 + \binom{3}{1}(1)^2(2x)^1 + \binom{3}{2}(1)(2x)^2 + \binom{3}{3}(2x)^3$

$= (1)(1) + (3)(1)(2x) + (3)(1)(4x^2) + (1)(8x^3)$

$= 1 + 6x + 12x^2 + 8x^3$

(vi) $(1 - 3x)^4 = \binom{4}{0}(1)^4 + \binom{4}{1}(1)^3(-3x)^1 + \binom{4}{2}(1)^2(-3x)^2 + \binom{4}{3}(1)^1(-3x)^3 + \binom{4}{4}(-3x)^4$

$= (1)(1) + (4)(1)(-3x) + (6)(1)(9x^2) + (4)(1)(-27x^3) + (1)(81x^4)$

$= 1 - 12x + 54x^2 - 108x^3 + 81x^4$

(vii) $(1 + 2x)^6 = \binom{6}{0}(1)^6 + \binom{6}{1}(1)^5(2x)^1 + \binom{6}{2}(1)^4(2x)^2 + \binom{6}{3}(1)^3(2x)^3 + \binom{6}{4}(1)^2(2x)^4$

$+ \binom{6}{5}(1)^1(2x)^5 + \binom{6}{6}(2x)^6$

$= (1)(1) + (6)(1)(2x) + (15)(1)(4x^2) + (20)(1)(8x^3) + (15)(1)(16x^4)$

$+ (6)(1)(32x^5) + (1)(64x^6)$

$= 1 + 12x + 60x^2 + 160x^3 + 240x^4 + 192x^5 + 64x^6$

(viii) $(1 + k)^7 = \binom{7}{0}(1)^7 + \binom{7}{1}(1)^6(k) + \binom{7}{2}(1)^5(k)^2 + \binom{7}{3}(1)^4(k)^3 + \binom{7}{4}(1)^3(k)^4$

$+ \binom{7}{5}(1)^2(k)^5 + \binom{7}{6}(1)^1(k)^6 + \binom{7}{7}(k)^7$

$= (1)(1) + (7)(1)(k) + (21)(1)(k^2) + (35)(1)(k^3) + (35)(1)(k^4)$

$+ (21)(1)(k^5) + (7)(1)(k^6) + (1)(k^7)$

$= 1 + 7k + 21k^2 + 35k^3 + 35k^4 + 21k^5 + 7k^6 + k^7$

Q. 2. (i) $(1+x)^4 = \binom{4}{0}(1)^4 + \binom{4}{1}(1)^3(x) + \binom{4}{2}(1)^2(x)^2 + \binom{4}{3}(1)(x)^3 + \binom{4}{4}(x)$

$= 1 + 4x + 6x^2 + 4x^3 + x^4$

(ii) $(1-x)^4 = 1 - 4x + 6x^2 - 4x^3 + x^4$ (letting $x = -x$)

Adding these gives:

$(1+x)^4 + (1-x)^4 = 2 + 12x^2 + 2x^4 = 2(1 + 6x^2 + x^4)$

(iii) Let $x = \sqrt{3}$ on both sides in (ii)

$(1+\sqrt{3})^4 + (1-\sqrt{3})^4 = 2(1 + 6(\sqrt{3})^2 + (\sqrt{3})^4)$

$= 2(1 + 18 + 9) = 56$

Q. 3. $(1+3x)^3 = \binom{3}{0}(1)^3 + \binom{3}{1}(1)^2(3x) + \binom{3}{2}(1)^1(3x)^2 + \binom{3}{3}(3x)^3$

$= (1)(1) + (3)(1)(3x) + (3)(1)(9x^2) + (1)(27x^3)$

$= 1 + 9x + 27x^2 + 27x^3$

Letting $x = 1$ on both sides:

LHS $= (1+3)^3 = (4)^3 = 64$

RHS $= 1 + 9 + 27 + 27 = 64 =$ LHS QED

Q. 4. $(1-2x)^5 = \binom{5}{0}(1)^5 + \binom{5}{1}(1)^4(-2x)^1 + \binom{5}{2}(1)^3(-2x)^2 + \binom{5}{3}(1)^2(-2x)^3$
$+ \binom{5}{4}(1)^1(-2x)^4 + \binom{5}{5}(-2x)^5$

$= (1)(1) + (5)(1)(-2x) + (10)(1)(4x^2) + (10)(1)(-8x^3) + (5)(1)(16x^4)$
$+ (1)(-32x^5)$

$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$

Letting $x = 2$ on both sides:

LHS $= (1-4)^5 = (-3)^5 = -243$

RHS $= 1 - 20 + 160 - 640 + 1{,}280 - 1{,}024$

$= -243 =$ LHS QED

Q. 5. $(1+x)^7 = 1 + 7x + 21x^2 + 35x^3 + 35x^4 + 21x^5 + 7x^6 + x^7$

$(1-x)^7 = 1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7$

Subtracting gives the result:

$(1+x)^7 - (1-x)^7 = 2(7x + 35x^3 + 21x^5 + x^7)$

Let $x = \sqrt{2}$ on both sides, gives

$(1+\sqrt{2})^7 - (1-\sqrt{2})^7 = 2(7\sqrt{2} + 35(\sqrt{2})^3 + 21(\sqrt{2})^5 + (\sqrt{2})^7)$

$= 2(7\sqrt{2} + 70\sqrt{2} + 84\sqrt{2} + 8\sqrt{2})$

$= 338\sqrt{2}$

Q. 6. $(x + \frac{1}{x})^6 = \binom{6}{0}(x^6) + \binom{6}{1}(x)^5\left(\frac{1}{x}\right)^1 + \binom{6}{2}(x)^4\left(\frac{1}{x}\right)^2 + \binom{6}{3}(x)^3\left(\frac{1}{x}\right)^3 + \binom{6}{4}(x)^2\left(\frac{1}{x}\right)^4 + \binom{6}{5}(x)^1\left(\frac{1}{x}\right)^5$
$+ \binom{6}{6}\left(\frac{1}{x}\right)^6$

$= 1 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$

The middle term is 20, which is a constant, independent of x.

Q. 7. (i) $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$

Adding gives:

$(a+b)^4 + (a-b)^4 = 2a^4 + 12a^2b^2 + 2b^4$

(ii) Let a become x and b become $\sqrt{x^2+1}$

$(x+\sqrt{x^2+1})^4 + (x-\sqrt{x^2+1})^4 = 2x^4 + 12x^2(x^2+1) + 2(x^2+1)^2$

$= 2x^4 + 12x^4 + 12x^2 + 2(x^4 + 2x^2 + 1)$

$= 16x^4 + 16x^2 + 2$

(iii) Let $x = 8$ on both sides, giving:

$(8+\sqrt{65})^4 + (8-\sqrt{65})^4 = 16(8)^4 + 16(8)^2 + 2 = 66{,}562$

Q. 8. (i) $(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

(ii) Let $a = 1$ and $b = 2x$, giving:

$(1+2x)^5 = 1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5$

$\therefore (1-2x)^5 = 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$

Subtracting gives:

$(1+2x)^5 - (1-2x)^5 = 2(10x + 80x^3 + 32x^5)$

(iii) Let $x = \sqrt{5}$ on both sides:

$(1+2\sqrt{5})^5 - (1-2\sqrt{5})^5 = 2(10\sqrt{5} + 80(5\sqrt{5}) + 32(25\sqrt{5}))$

$= 2{,}420\sqrt{5} = n\sqrt{5}$

$\therefore n = 2{,}420$

Exercise 1.6

Q. 1. (i) $\binom{8}{2}(1)^6(x)^2 = 28x^2$

(ii) $\binom{7}{2}(1)^3(2x)^4 = (35)(1)(16x^4) = 560x^4$

(iii) $\binom{9}{3}(x)^6(y)^3 = 84x^6y^3$

(iv) $\binom{8}{6}(2)^2(-x)^6 = (28)(4)(x^6) = 112x^6$

(v) $\binom{6}{3}(x)^3(y)^3 = 20x^3y^3$

(vi) $\binom{6}{5}(7x)^1(-y)^5 = (6)(7x)(-y^5) = -42xy^5$

(vii) $\binom{7}{6}(3)^1(-2x)^6 = (7)(3)(64x^6) = 1{,}344x^6$

(viii) $\binom{10}{5}(1)^5(-2x)^5 = (252)(1)(-32x^5) = -8{,}064x^5$

(ix) $\binom{5}{4}(1)^1(x^2)^4 = (5)(1)(x^8) = 5x^8$

(x) $\binom{11}{2}(1)^9(0.2)^2 = (55)(1)(0.04) = 2.2$

Q. 2. (i) $\binom{10}{4}(1)^6(x)^4 = 210x^4$: Answer: 210

(ii) $\binom{5}{4}(2)^1(x)^4 = 10x^4$: Answer: 10

(iii) $\binom{6}{2}(1)^4(x^2)^2 = 15x^4$: Answer: 15

(iv) $\binom{8}{4}(1)^4(-3x)^4 = 5{,}670x^4$: Answer: 5,670

(v) $\binom{3}{2}(2)^1(\sqrt{2}x^2)^2 = 12x^4$: Answer: 12

Q. 3. $\binom{9}{3}(p)^6(q)^3 = 84p^6q^3 = (84)\left(\frac{1}{2}\right)^6(2)^3 = 10.5$

Q. 4. $\binom{10}{0}(1)^{10} + \binom{10}{1}(1)^9(0.01)^1 + \binom{10}{2}(1)^8(0.01)^2$

$= 1 + 0.1 + 0.0045$

$= 1.1045$

Q. 5. $\binom{9}{2}(a)^7(2b)^2 = (36)a^7(4b^2) = 144a^7b^2$

$\therefore$ coefficient of $a^7 = 144b^2$

Q. 6. $\binom{12}{6}(x)^6(-2y)^6 = (924)(x^6)(64y^6)$

$= 59{,}136x^6y^6$

$= 59{,}136\left(\frac{3}{2}\right)^6\left(\frac{1}{6}\right)^6$

$= 14.4375$

Q. 7. (i) $\binom{7}{3}(1)^4(-5x)^3 = (35)(1)(-125x^3)$

$= -4{,}375x^3$

Answer: $-4{,}375$

(ii) $\binom{7}{4}(1)^3(-5(0.2))^4 = 35$

Q. 8. (i) $\binom{4}{r}(x)^{4-r}\left(\frac{1}{x}\right)^r$ = General term

Power of $r = \frac{x^{4-r}}{x^r} = x^{4-2r}$

$\therefore 4 - 2r = 0$

$\therefore r = 2$

Independent term $= \binom{4}{2}(x)^2\left(\frac{1}{x^2}\right) = 6$

(ii) $\binom{6}{r}(x)^{6-r}\left(-\frac{2}{x}\right)^r$ = General term

Power of $x = \frac{x^{6-r}}{x^r} = x^{6-2r}$

$\therefore 6 - 2r = 0$

$\therefore r = 3$

Independent term $= \binom{6}{3}(x)^3\left(-\frac{2}{x}\right)^3 = -160$

(iii) General term $= \binom{10}{r}(x^3)^{10-r}\left(\frac{1}{x^2}\right)^r$

Power of $x = \frac{(x^3)^{10-r}}{x^{2r}} = \frac{x^{30-3r}}{x^{2r}} = x^{30-5r}$

$\therefore 30 - 5r = 0$

$\therefore r = 6$

Independent term $= \binom{10}{6}(x^3)^4\left(\frac{1}{x^2}\right)^6$

$= 210$

(iv) General term $= \binom{8}{r}(2x^3)^{8-r}\left(-\frac{1}{3x}\right)^r$

Power of $x = \frac{(x^3)^{8-r}}{x^r} = \frac{x^{24-3r}}{x^r} = x^{24-4r}$

$\therefore 24 - 4r = 0$

$\therefore r = 6$

Independent term $= \binom{8}{6}(2x^3)^2\left(-\frac{1}{3x}\right)^6$

$= (28)(4x^6)\left(\frac{1}{729x^6}\right) = \frac{112}{729}$

(v) $\binom{15}{r}(x^2)^{15-r}\left(\frac{1}{\sqrt{x}}\right)^r$ = General term

Power of $x = \frac{(x^2)^{15-r}}{(\sqrt{x})^r} = \frac{x^{30-2r}}{x^{\frac{1}{2}r}} = x^{30-2.5r}$

$\therefore 30 - 2.5r = 0$

$\therefore r = 12$

Independent term $= \binom{15}{12}(x^2)^3\left(\frac{1}{\sqrt{x}}\right)^{12} = 455$

Q. 9. (i) General term $= \binom{10}{r}(1)^{10-r}(-7x)^r$

$\therefore r = 1$ (to give x^1)

Term $= \binom{10}{1}(1)^9(-7x)^1 = -70x$

$\therefore$ Coefficient of $x = -70$

(ii) General term $= \binom{7}{r}(3x^3)^{7-r}\left(\frac{2}{x^2}\right)^r$

Power of $x = \frac{(x^3)^{7-r}}{x^{2r}} = \frac{x^{21-3r}}{x^{2r}} = x^{21-5r}$

$\therefore 21 - 5r = 1$

$\therefore r = 4$

Term $= \binom{7}{4}(3x^3)^3\left(\frac{2}{x^2}\right)^4$

$= (35)(27x^9)\left(\frac{16}{x^8}\right)$

$= 15{,}120x$

Coefficient of $x = 15{,}120$

Q. 10. (i) General term $= \binom{10}{r}(1)^{10-r}(-3x)^r$

The power of $x = x^r$

$\therefore r = 2$

Term $= \binom{10}{2}(1)^8(-3x)^2 = 405x^2$

Coefficient of $x^2 = 405$

(ii) General term $= \binom{10}{r}(x)^{10-r}\left(-\frac{1}{x^3}\right)^r$

The power of $x = \frac{x^{10-r}}{(x^3)^r} = \frac{x^{10-r}}{x^{3r}} = x^{10-4r}$

$\therefore 10 - 4r = 2$

$\therefore r = 2$

Term $= \binom{10}{2}(x)^8\left(\frac{-1}{x^3}\right)^2 = 45x^2$

Coefficient of $x^2 = 45$

Q. 11. $\binom{6}{3}(k)^3(2x)^3 = 4{,}320x^3$

$\therefore (20)(k^3)(8x^3) = 4{,}320x^3$

$\therefore 160k^3 = 4{,}320$

$\therefore k^3 = 27$

$\therefore k = 3$

Q. 12. $t_3 = \binom{10}{2}(8)^8(kx)^2 = (45)(8)^8(k^2x^2)$

Coefficient = $45k^2(8^8)$

$t_4 = \binom{10}{3}(8)^7(kx)^3 = (120)(8^7)(k^3x^3)$

Coefficient = $120k^3(8^7)$

$\therefore 45k^2(8^8) = 120k^3(8^7)$

Divide both sides by $k^2(8^7)$:

$\therefore 45(8) = 120(k)$

$\therefore k = 3$

Q. 13. $t_2 = t_3$

$\therefore \binom{6}{1}(1)^5(x)^1 = \binom{6}{2}(1)^4(x)^2$

$\therefore 6x = 15x^2$

$\therefore 2x = 5x^2$

$\therefore 5x^2 - 2x = 0$

$\therefore x(5x - 2) = 0$

$\therefore x = 0 \quad \text{or} \quad x = 0.4$

The only positive value is $x = 0.4$

Q. 14. $t_{r+1} = \binom{12}{r}(x)^{12-r}\left(\frac{1}{x}\right)^r$

$= \binom{12}{r}\frac{x^{12-r}}{x^r} = \binom{12}{r}x^{12-2r}$

$t_5 > t_4$

$\therefore \binom{12}{4}x^{12-8} > \binom{12}{3}x^{12-6}$

$\therefore 495x^4 > 220x^6$ (Divide by x^4, which is positive)

$\therefore 2.25 > x^2$

$\therefore -1.5 < x < 1.5$

Q. 15. $(1 + ax)^n = \binom{n}{0}(1)^n + \binom{n}{1}(1)^{n-1}(ax)$

$+ \binom{n}{2}(1)^{n-2}(ax)^2 + \cdots$

$= (1)(1) + (n)(1)(ax) + \frac{n(n-1)}{2}(1)(a^2x^2) + \cdots$

$= 1 + nax + \frac{n(n-1)a^2}{2}x^2 + \cdots$ } IDENTITY

$= 1 + 24x + 240x^2 + \cdots$

$\therefore na = 24 \quad \text{and} \quad \frac{n(n-1)a^2}{2} = 240$

$\therefore a = \frac{24}{n} \quad \text{and} \quad n(n-1)a^2 = 480$

$\therefore n(n-1)\left(\frac{24}{n}\right)^2 = 480$

$\therefore n(n-1)\left(\frac{576}{n^2}\right) = 480$

$\therefore \frac{576(n-1)}{n} = 480$

$\therefore 576(n-1) = 480n$

$\therefore 12(n-1) = 10n$

$\therefore 12n - 12 = 10n$

$\therefore 2n = 12$

$\therefore n = 6$

$\therefore a = 4$

Q. 16. A similar argument as in Q15 leads to:

$nk = -20 \quad \text{and} \quad \frac{n(n-1)k^2}{2} = 180$

$\therefore k = \frac{-20}{n} \quad \text{and} \quad n(n-1)k^2 = 360$

$\therefore n(n-1)\left(\frac{-20}{n}\right)^2 = 360$

$\therefore n(n-1)\left(\frac{400}{n^2}\right) = 360$

$\therefore \frac{400(n-1)}{n} = 360$

$\therefore 400(n-1) = 360n$

$\therefore 10(n-1) = 9n$

$\therefore 10n - 10 = 9n$

$\therefore n = 10$

$\therefore k = \frac{-20}{n} = \frac{-20}{10} = -2$

The 4th term of $(1 - 2x)^{10}$ is:

$\binom{10}{3}(1)^7(-2x)^3 = 120(1)(-8x^3) = -960x^3$

$\therefore p = -960$

Q. 17. The general term of $((a + b) + c)^8$ is $t_{r+1} = \binom{8}{r}(a + b)^{8-r}(c)^r$

We require c^2:

$\therefore r = 2$

$\therefore t_{r+1} = \binom{8}{2}(a + b)^6c^2 = 28c^2(a + b)^6$

But this is a series of terms. We require the term with a^5bc^2. This will be:

$28c^2\binom{6}{1}(a^5)(b)^1 = 168a^5bc^2$

$\therefore$ the coefficient of $a^5bc^2 = 168$

Q. 18. The general term in $[(x + y) + z]^6$

is $t_{r+1} = \binom{6}{r}(x + y)^{6-r}(z)^r$

We require z^1.

$\therefore\ r = 1$

$t_{r+1} = \binom{6}{1}(x + y)^5 z^1 = \binom{6}{1}z(x + y)^5$

This is a series of terms.

We require the term in x^3y^2z.

It will be $\binom{6}{1}z\binom{5}{2}(x)^3y^2$

$= \binom{6}{1}\binom{5}{2}\, x^3y^2z$

The coefficient $= \binom{6}{1}\binom{5}{2} = \frac{6!}{1!5!} \times \frac{5!}{2!3!} = \frac{6!}{3!2!1!}$ QED

Q. 19. Take $[(a + b) + (c + d)]^7$

We need $\binom{7}{2}(a + b)^5(c + d)^2$ to get a^3b^2 and cd. Of these, we require:

$\binom{7}{2}\binom{5}{2}(a^3)(b^2)\binom{2}{1}(c^1)(d^1)$

$= (21)(10)(2)a^3b^2cd$

$= 420a^3b^2cd$

The coefficient is 420.

Q. 20. $\binom{5}{0}(1)^5 + \binom{5}{1}(1)^4(2x + 3x^2)^1 + \binom{5}{2}(1)^3(2x + 3x^2)^2 + \cdots +$ terms with x^3 or higher powers

$= (1)(1) + (5)(1)(2x + 3x^2) + (10)(1)(2x + 3x^2)^2 +$ higher powers

$= 1 + 10x + 15x^2 + 10(4x^2 +$ higher powers$)$

$= 1 + 10x + 55x^2 +$ higher powers

The coefficient of x^2 is 55.

Q. 21. Take $(1 + (x + x^2))^6$.

The only terms that can have a term in x^3 are the following:

$\binom{6}{2}(1)^4(x + x^2)^2 + \binom{6}{3}(1)^3(x + x^2)^3$

$= (15)(1)(x^2 + 2x^3 + x^4) + (20)(1)(x^3 +$ higher powers$)$

The only terms in x^3 are $30x^3 + 20x^3 = 50x^3$.

The coefficient of x^3 is 50.

Q. 22. Take $(1 + (2x + x^2))^4$

The only terms that contain x^4 are:

$\binom{4}{2}(1)^2(2x + x^2)^2 + \binom{4}{3}(1)(2x + x^2)^3 + \binom{4}{4}(1)(2x + x^3)^4$

$= 6(4x^2 + 4x^3 + x^4) + 4(8x^3 + 12x^4 + 6x^5 + x^6) + 16x^4 +$ higher powers

The terms in $x^4 = 6x^4 + 48x^4 + 16x^4 = 70x^4$

The coefficient is 70.

Exercise 1.7

Q. 1.

$$x^2 + 4x + 1$$
$$x - 2 \,\big|\, x^3 + 2x^2 - 7x - 2$$
$$\ominus x^3 \oplus 2x^2$$
$$4x^2 - 7x$$
$$\ominus 4x^2 \oplus 8x$$
$$x - 2$$
$$\ominus x \oplus 2$$
$$0$$

Q. 2.

$$x^2 + 3x - 10$$
$$3x + 4 \,\big|\, 3x^3 + 13x^2 - 18x - 40$$
$$\ominus 3x^3 \ominus 4x^2$$
$$9x^2 - 18x$$
$$\oplus 9x^2 \ominus 12x$$
$$-30x - 40$$
$$\oplus 30x \oplus 40$$
$$0$$

Q. 3.

$$2x^2 - 9x + 4$$
$$3x - 1 \,\big|\, 6x^3 - 29x^2 + 21x - 4$$
$$\ominus 6x^3 \oplus 2x^2$$
$$-27x^2 + 21x$$
$$\oplus 27x^2 \ominus 9x$$
$$12x - 4$$
$$\ominus 12x \oplus 4$$
$$0$$

Q. 4.

$$2x^2 + 5x$$
$$7x - 1 \,\big|\, 14x^3 + 33x^2 - 5x$$
$$\ominus 14x^3 \oplus 2x^2$$
$$35x^2 - 5x$$
$$35x^2 - 5x$$
$$0$$

Q. 5.

$$12x^2 + 8x - 4$$
$$3x - 2 \,\big|\, 36x^3 \quad - 28x + 8$$
$$\ominus 36x^3 \oplus 24x^2$$
$$24x^2 - 28x$$
$$\ominus 24x^2 \oplus 16x$$
$$-12x + 8$$
$$\oplus 12x \ominus 8$$
$$0$$

Q. 6.

$$3x^3 - 4x^2 - 13x + 14$$
$$5x + 3 \,\big|\, 15x^4 - 11x^3 - 77x^2 + 31x + 42$$
$$\ominus 15x^4 \ominus 9x^3$$
$$-20x^3 - 77x^2$$
$$\oplus 20x^3 \oplus 12x^2$$
$$-65x^2 + 31x$$
$$\oplus 65x^2 \oplus 39x$$
$$70x + 42$$
$$\ominus 70x \ominus 42$$
$$0$$

Q. 7.

$$x^2 - x - 12$$
$$x + 1 \,\big|\, x^3 + 0x^2 - 13x - 12$$
$$-(x^3 + x^2)$$
$$-x^2 - 13x$$
$$-(-x^2 - x)$$
$$-12x - 12$$
$$-12x - 12$$
$$0$$

$x^2 - x - 12$

$= (x - 4)(x + 3)$

Q. 8.

$$2x^3 - 12x^2 + 10x$$
$$x - 1 \,\big|\, 2x^4 - 14x^3 + 22x^2 - 10x$$
$$\ominus 2x^4 \oplus 2x^3$$
$$-12x^3 + 22x^2$$
$$\oplus 12x^3 \ominus 12x^2$$
$$10x^2 - 10x$$
$$10x^2 - 10x$$
$$0$$

Q. 9.

$$
\begin{array}{r|l}
 & 9x^2 - 16 \\
\hline
4x^2 - 25 & 36x^4 - 289x^2 + 400 \\
 & \ominus 36x^4 \oplus 225x^2 \\
\hline
 & -64x^2 + 400 \\
 & \oplus 64x^2 \oplus 400 \\
\hline
 & 0
\end{array}
$$

Answer: $= (2x - 5)(3x - 4)(2x + 5)(3x + 4)$

Q. 10.

$$
\begin{array}{r|l}
 & x^3 - 8 \\
\hline
x^2 - x - 6 & x^5 - x^4 - 6x^3 - 8x^2 + 8x + 48 \\
 & -\ (x^5 - x^4 - 6x^3) \\
\hline
 & 0 - 8x^2 + 8x + 48 \\
 & -\ (-8x^2 + 8x + 48) \\
\hline
 & 0
\end{array}
$$

$x^3 - 8 = (x - 2)(x^2 + 2x + 4)$

Answer: $= (x - 3)(x + 2)(x - 2)(x^2 + 2x + 4)$

Q. 11.

$$
\begin{array}{r|l}
 & 3x^2 - 8x \\
\hline
2x + 1 & 6x^3 - 13x^2 - 19x + 12 \\
 & \ominus 6x^3 \overset{\ominus}{+} 3x^2 \\
\hline
 & -16x^2 - 19x \\
 & \oplus 16x^2 \oplus 8x \\
\hline
 & -11x + 12
\end{array}
$$

Doesn't divide in evenly $\therefore$ not a factor

Q. 12.

$$
\begin{array}{r|l}
 & -2x^3 - 7x^2 + 4x \\
\hline
3x + 4 & -6x^4 - 29x^3 - 16x^2 + 16x \\
 & \oplus 6x^4 \oplus 8x^3 \\
\hline
 & -21x^3 - 16x^2 \\
 & \oplus 21x^3 \oplus 28x^2 \\
\hline
 & 12x^2 + 16x \\
 & \ominus 12x^2 \overset{\ominus}{+} 16x \\
\hline
 & 0
\end{array}
$$

$3x + 4$ is a factor. $5x + 4$ is not a factor, as it doesn't divide into $-6x^4 - 29x^3 - 16x^2 + 16x$ without a remainder.

Revision Exercises

Q. 1. (a) (i) $4x(x - 3) + 2(x - 5)$

$= 4x^2 - 12x + 2x - 10$

$= 4x^2 - 10x - 10$

(ii) $3a(a^2 + b - 2c) - 2b(a^2 + b - 2c)$

$= 3a^3 + 3ab - 6ac - 2a^2b - 2b^2 + 4bc$

(iii) $(x + 5)(x + 1)$

$= x(x + 1) + 5(x + 1)$

$= x^2 + x + 5x + 1$

$= x^2 + 6x + 1$

(iv) $(5x - 2)(3x - 4)$

$= 5x(3x - 4) - 2(3x - 4)$

$= 15x^2 - 20x - 6x + 8$

$= 15x^2 - 26x + 8$

(v) $(11p - 3q)(11p + 3q)$

$= 11p(11p - 3q) - 3q(11p + 3q)$

$= 121p^2 + 33pq - 33pq - 9q^2$

$= 121p^2 - 9q^2$

(b) (i) $9x^2 + 42x + 49$

(ii) $8x^3 - 12x^2 + 6x - 1$

(iii) $64x^3 - 240x^2 + 300x - 125$

(iv) $(4p + 3)^3(p - 2)$

$= [64p^3 + 3(16p^2)(3) + 3(4p)(9) + 27][p - 2]$

$= 64p^4 + 144p^3 + 108p^2 + 27p - 128p^3 - 288p^2 - 216p - 54$

$= 64p^4 + 16p^3 - 180p^2 - 189p - 54$

(c) (i) $1(2a)^5 + 5(2a)^4 + 10(2a)^3 + 10(2a)^2 + 5(2a) + 1$

$= 32a^5 + 80a^4 + 80a^3 + 40a^2 + 10a + 1$

(ii) $1(4b)^4 + 4(4b)^3(7c) + 6(4b)^2(7c)^2 + 4(4b)(7c)^3 + 1(7c)^4$

$= 256b^4 - 1{,}792b^3c + 4{,}704b^2c^2 - 5{,}488bc^3 + 2{,}401c^4$

(iii) $1(6x)^6 + 6(6x)^5(-5)^1 + 15(6x)^4(-5)^2 + 20(6x)^3(-5)^3 + 15(6x)^2(-5)^4 + 6(6x)(-5)^5 + 1(-5)^6$

$= 46{,}656x^6 - 233{,}280x^5 + 486{,}000x^4 - 540{,}000x^3 + 337{,}500x^2 - 112{,}500x + 15{,}625$

Q. 2. (a) (i) $(x + 9)(x - 10)$

(ii) $15ac - 12ad - 10bc + 8bd$

$= 3a(5c - 4d) - 2b(5c - 4d)$

$= (3a - 2b)(5c - 4d)$

(iii) $(2x - 9)(2x + 9)$

(iv) $(2x + 1)(2x + 1)$

(b) (i) $(5x - 7y)(5x + 7y)$

(ii) $(11a - 12b)(11a + 12b)$

(iii) $6m^2 + 15bx - 10bm - 9mx$

$= 6m^2 - 10bm - 9mx + 15bx$

$= 2m(3m - 5b) - 3x(3m - 5b)$

$= (2m - 3x)(3m - 5b)$

(iv) $(5x + 2)(2x - 1)$

(v) $(7x - 4)(2x - 1)$

(c) (i) $x^3 + 27$

$(x + 3)(x^2 - 3x + 9)$

(ii) $2b^3 + 2{,}000$

$2\,(b^2 + 1{,}000)$

$2\,(b + 10)\,(b^2 - 10b + 100)$

(iii) $y^3 - 1$

$(y - 1)(y^2 + y + 1)$

(iv) $8y^3 - 1$

$(2y - 1)(4y^2 + 2y + 1)$

Q. 3. (a) (i) $\dfrac{3x + 12 + 5x - 15}{(x - 3)(x + 4)}$

$= \dfrac{8x - 3}{(x - 3)(x + 4)}$

(ii) $\dfrac{14y - 7 - 12y - 6}{4y^2 - 1}$

$= \dfrac{2y - 13}{4y^2 - 1}$

(iii) $\dfrac{2x - 2 - x^2 - x}{x^2 - 1}$

$= \dfrac{x - 2 - x^2}{x^2 - 1}$

(b) (i) $\dfrac{3\cancel{(a - 3b)}}{6\cancel{(a - 3b)}}$

$= \dfrac{1}{2}$

(ii) $\dfrac{2\cancel{(4x - 5)}}{\cancel{(4x - 5)}(4x + 5)}$

$= \dfrac{2}{4x + 5}$

(iii) $\dfrac{\cancel{(a - b)}(a^2 + ab + b^2)}{\cancel{(a - b)}6} = \dfrac{(a^2 + ab + b^2)}{6}$

(iv) $\dfrac{b(y + 1) - 1(y + 1)}{(b - 1)(b^2 + b + 1)}$

$= \dfrac{y + 1}{b^2 + b + 1}$

(v) $\dfrac{x(x - 1)}{(x - 1)(x - 3)}$

$= \dfrac{x}{x - 3}$

(c) (i) $\dfrac{5 - 1}{x - 2} = \dfrac{4}{x - 2}$

(ii) $\dfrac{7 - 5}{2y - 1} = \dfrac{2}{2y - 1}$

(iii) $\dfrac{2 - 2}{b - a} = 0$

(iv) $\dfrac{9 + 4}{2x - 1} = \dfrac{13}{2x - 1}$

Q. 4. (a) (i) $\dfrac{\cancel{(2x + 1)}(x + 4)}{\cancel{(2x + 1)}(x + 5)} = \dfrac{x + 4}{x + 5}$

(ii) $\dfrac{\cancel{(a - b)}(a + b)}{\cancel{(a - b)}(a^2 + ab + b^2)}$

$= \dfrac{a + b}{a^2 + ab + b^2}$

(iii) $\dfrac{2\cancel{(x - y)}}{-3\cancel{(x - y)}}$

$= -\dfrac{2}{3}$

(iv) $\dfrac{x - 3}{(3 - x)(3 + x)}$

$= \dfrac{-\cancel{(3 - x)}}{\cancel{(3 - x)}(3 + x)} = \dfrac{-1}{x + 3}$

(b) (i) $\dfrac{6x - x + 3}{(x - 3)(x + 3)}$

$= \dfrac{5x + 3}{(x - 3)(x + 3)}$

(ii) $\dfrac{4(3y + 2) - 2y}{(3y - 2)(3y + 2)}$

$= \dfrac{12y + 8 - 2y}{9y^2 - 4}$

$= \dfrac{10y + 8}{9y^2 - 4}$

(c) (i) $\dfrac{(x-y)^2 - z^2}{x^2 - (y+z)^2}$

$= \dfrac{(x-y-z)(x-y+z)}{(x-y-z)(x+y+z)}$

$= \dfrac{x-y+z}{x+y+z}$

(ii) $\dfrac{(x-5)(x+5)(x)(x-8)}{(x-8)(x+8)(x-5)}$

$= \dfrac{x(x+5)}{x+8}$

Q. 5. (a) (i)

$$\begin{array}{r} 4x^2 - x - 3 \\ x-1 \,\big)\, 4x^3 - 5x - 2x + 3 \\ -(4x^3 - 4x^2) \\ \hline -x^2 - 2x \\ -(-x^2 + x) \\ \hline -3x + 3 \\ -(-3x+3) \\ \hline 0 \end{array}$$

(ii)

$$\begin{array}{r} 12x^2 - 2x - 4 \\ 3x-1 \,\big)\, 36x^3 - 18x^2 - 10x + 4 \\ -(36x^3 - 12x^2) \\ \hline -6x^2 - 10x \\ -(-6x^2 + 2x) \\ \hline -12x + 4 \\ -(-12x + 4) \\ \hline 0 \end{array}$$

(iii)

$$\begin{array}{r} 8x^3 - 10x^2 + x + 1 \\ 3x+1 \,\big)\, 24x^4 - 22x^3 - 7x^2 + 4x + 1 \\ -(24x^4 + 8x^3) \\ \hline -30x^3 - 7x^2 \\ -(30x^3 - 10x^2) \\ \hline 3x^2 + 4x \\ -(x^2 + x) \\ \hline 3x + 1 \\ -(3x+1) \\ \hline 0 \end{array}$$

(iv)

$$\begin{array}{r} x^3 + 5x^2 + 2x - 8 \\ 2x+3 \,\big)\, 2x^4 + 13x^3 + 19x^2 - 10x - 24 \\ -(2x^4 + 3x^3) \\ \hline 10x^3 + 19x^2 \\ -(10x^3 + 15x^2) \\ \hline 4x^2 - 10x \\ -(4x^2 + 6x) \\ \hline -16x - 24 \\ -(-16x + 24) \\ \hline 0 \end{array}$$

(b) (i)

$$\begin{array}{r} 6x^2 + 7x - 5 \\ 8x+1 \,\big)\, 48x^3 + 62x^2 - 33x - 5 \\ -(48x^3 + 6x^2) \\ \hline 56x^2 - 33x \\ -(56x^2 + 7x) \\ \hline -40x - 5 \\ -(-40x - 5) \\ \hline 0 \end{array}$$

Remainder = 0 $\therefore$ factor

Other factors: $(3x+5)(2x-1)$

(ii)

$$\begin{array}{r} 8x^2 + 38x + 69 \\ x-2 \,\big)\, 8x^3 + 22x^2 - 7x - 3 \\ -(8x^3 - 16x^2) \\ \hline 38x^2 - 7x \\ -(38x^2 - 76x) \\ \hline 69x - 3 \\ -(69x + 138) \\ \hline 135 \end{array}$$

Remainder $\neq 0$ $\therefore$ not a factor

(c) $\dfrac{2x(x-3) + 3x(x+3) - 5x^2 - 9}{x^2 - 9}$

$= \dfrac{2\cancel{x^2} - 6x + 3\cancel{x^2} + 9x - 5\cancel{x^2} - 9}{x^2 - 9}$

$= \dfrac{3x - 9}{(x-3)(x+3)}$

$= \dfrac{3\cancel{(x-3)}}{\cancel{(x-3)}(x+3)}$

$= \dfrac{3}{x+3}$

Q. 6. (a) (i) $(2x + 5)(4x^2 - 10x + 25)$

(ii) $(x - 6)(x^2 + 6x + 36)$

(iii) $(5x + 6y + x + y)(5x + 6y - x - y)$

$= (6x + 7y)(4x + 5y)$

(iv) $ax + ay + bx + by$

$= a(x + y) + b(x + y)$

$= (a + b)(x + y)$

(v) $(3x - 2y)(3x - 2y)$

(b) (i) $\binom{7}{3}(3)^4(-2x)^3 = (35)(81)(-8x^3)$

$= -22{,}680x^3$

The coefficient of x^3 is $-22{,}680$.

(ii) The general term $= \binom{9}{r}(x)^{9-r}\left(\frac{-2}{x^2}\right)^r$

Extract the power of x: $\frac{x^{9-r}}{x^{2r}} = x^{9-3r}$

$9 - 3r = 3$

$r = 2$

General term $= \binom{9}{2}(x)^7\left(\frac{-2}{x^2}\right)^2$

$= 36(x^7)\left(\frac{4}{x^4}\right) = 144x^3$

The coefficient of x^3 is 144.

(c) $(1 + x)^4 = \binom{4}{0}(1)^4 + \binom{4}{1}(1)^3(x)^1$

$+ \binom{4}{2}(1)^2(x)^2 + \binom{4}{1}(1)^1(x)^3 + \binom{4}{1}(x)^4$

$= 1 + 4x + 6x^2 + 4x^3 + x^4$

$\therefore (1 + \sqrt{2})^4 = 1 + 4(\sqrt{2}) + 6(\sqrt{2})^2 + 4(\sqrt{2})^3 + (\sqrt{2})^4$

$= 1 + 4\sqrt{2} + 12 + 8\sqrt{2} + 4$

$= 17 + 12\sqrt{2}$

(d) (i) $\frac{a(x + y) - c(x + y)}{a(x + y) + c(x + y)}$

$= \frac{(a - c)\cancel{(x + y)}}{(a + c)\cancel{(x + y)}} = \frac{(a - c)}{(a + c)}$

(ii) $\frac{9y^3 - y}{3y^2 + 8y - 3} = \frac{y\,\cancel{(3y - 1)}(3y + 1)}{(y + 3)\cancel{(3y - 1)}}$

$= \frac{y(3y + 1)}{y + 3}$

(iii) $\frac{(a + b - c)\cancel{(a + b + c)}}{(a - b - c)\cancel{(a + b + c)}}$

$= \frac{a + b - c}{a - b - c}$

Q. 7. (a) (i) $(m^2 + n^2)^2 = (m^2 - n^2)^2 + (2mn)^2$

$m^4 + 2m^2n^2 + n^4 = m^4 - 2m^2n^2 + n^4 + 4m^2n^2$

$m^4 + 2m^2n^2 + n^4 = m^4 + 2m^2n^2 + n^4$

LHS = RHS

(ii) $m = 5$ Hyp $= 25 + 4 = 29$

$n = 2$ $25 - 4 = 21$

$2(5)(2) = 20$

(b) The general term $= \binom{10}{r}(x^2)^{10-r}\left(\frac{3}{x}\right)^r$

Extract the power of x: $\frac{x^{20-2r}}{x^r} = x^{20-3r}$

$20 - 3r = 2$

$18 = 3r$

$r = 6$

General term $= \binom{10}{6}(x^2)^4\left(\frac{3}{x}\right)^6$

$= 210(x^8)\left(\frac{729}{x^6}\right) = 153{,}090x^2$

The coefficient of x^2 is 153,090.

(c) (i) $3(x^2 - 25)$

$= 3(x - 5)(x + 5)$

(ii) $x(9x^2 - 25)$

$= x(3x - 5)(3x + 5)$

(iii) $(x^2 - y^2)(x^2 + y^2)$

$= (x^2 + y^2)(x - y)(x + y)$

(iv) $(x^2 - 9)(x^2 + 9)$

$= (x^2 + 9)(x - 3)(x + 3)$

(v) $(a - b)(x^2) + (a - b)(-y^2)$

$= (a - b)(x - y)(x + y)$

Q. 8. (a) (i) $x^2 + 20x + 51$

$= (x + 3)(x + 17)$

(ii) $-1(x^2 - 169)$

$= -1(x - 13)(x + 13)$

(iii) $a^2 - 2ab + b^2$

$= (a - b)(a - b)$

(iv) $a^2 - 2ab + b^2 - c^2$

$= (a - b)^2 - c^2$

$= (a - b - c)(a - b + c)$

(b) (i)

$$\begin{array}{r|l} & 4x^3 - 16x^2 - 23 \ + 9 \\ \hline x-1 & 4x^4 - 20x^3 - 7x^2 + 32x + 15 \\ & \ominus 4x^4 \ominus + \ 4x^3 \\ \hline & \quad - 16x^3 - \ 7x^2 \\ & \quad \oplus 16x^3 \ominus + 16x^2 \\ \hline & \qquad - 23x^2 + 32x \\ & \qquad \oplus 23x^2 \oplus 23x \\ \hline & \qquad\qquad 9x + 15 \\ & \qquad\qquad 9x - 9 \\ \hline \end{array}$$

Remainder exists ∴ not a factor

(ii)

$$\begin{array}{r|l} & 3x^2 + \ x \ - 14 \\ \hline 4x^2 - 1 & 12x^4 + 4x^3 - 59x^2 - x + 14 \\ & -(12x^4 + 0x^3 - \ 3x^2) \\ \hline & \quad 4x^3 - 56x^2 - x \\ & \quad -(4x^3 + \ 0x^2 - x) \\ \hline & \qquad -56x^2 \quad + 14 \\ & \qquad -(-56x^2 \quad + 14) \\ \hline & \qquad\qquad 0 \end{array}$$

$3x^2 + x - 14$

$= (3x + 7)(x - 2)$

Therefore, the other factors are $x - 2$ and $3x + 7$.

(iii)

$$\begin{array}{r|l} & 4x^2 + 4x \ - 3 \\ \hline x^3 - x^2 & 4x^5 + 0x^4 - 7x^3 + 3x^2 \\ & -(4x^5 - 4x^4) \\ \hline & \quad 4x^4 - 7x^3 \\ & \quad -(4x^4 - 4x^3) \\ \hline & \qquad -3x^3 + 3x^2 \\ & \qquad -(-3x^3 + 3x^2) \\ \hline & \qquad\qquad 0 \end{array}$$

$4x^2 + 4x - 3$

$= (2x + 3)(2x - 1)$

Therefore, the other factors are $2x + 3$ and $2x - 1$.

(c) (i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(ii) $(x + y)(x^2 - xy + y^2) + 3x^2y + 3xy^2$

$= x^3 - \cancel{x^2y} + \cancel{xy^2} + \cancel{x^2y} - \cancel{xy^2} + y^3 + 3x^2y + 3xy^2$

$= x^3 + 3x^2y + 3y^2 + y^3$

$= (x + y)^3$

(iii) $(x + y)^3 + z^3$

Answer: $= (x + y + z)[(x + y)^2 - (x + y)(z) + z^2)]$

$= (x + y + z)(x^2 + y^2 + z^2 + 2xy - xz - yz)$

Q. 9. (a) (i) $\dfrac{2x^2 + 3x - 20}{x + 4}$

$= \dfrac{(2x - 5)\cancel{(x + 4)}}{\cancel{x + 4}}$

Breadth $= 2x - 5$

(ii)

$$\begin{array}{r|l} & \qquad 3x - 1 \\ \hline 2x^2 + 3x - 20 & 6x^3 \ + 7x^2 \ - \ 63x + 20 \\ & \ominus \ 6x^3 \oplus 9x^2 \oplus 60x \\ \hline & \qquad -2x^2 - \ 3x + 20 \\ & \qquad \oplus 2x^2 \oplus 3x \oplus 20 \\ \hline & \qquad\qquad 0 \end{array}$$

Height $= 3x - 1$

(b) $t_4 = \binom{11}{3}(x)^8\left(\dfrac{2}{x}\right)^3 = 165(x^8)\left(\dfrac{8}{x^3}\right) = 1320x^5$

$t_5 = \binom{11}{4}(x)^7\left(\dfrac{2}{x}\right)^4 = 330(x^7)\left(\dfrac{16}{x^4}\right) = 5280x^3$

$t_4 < t_5 \Rightarrow 1320x^5 < 5280x^3$

$\therefore x^2 < 4$ as $x > 0$

$\therefore 0 < x < 2$

(c) $\dfrac{\dfrac{z(z + 1) + z(z - 1)}{\cancel{z^2 - 1}}}{\dfrac{z(z + 1) - z(z - 1)}{\cancel{z^2 - 1}}}$

$= \dfrac{z^2 + \cancel{z} + z^2 - \cancel{z}}{\cancel{z^2} + z - \cancel{z^2} + z}$

$= \dfrac{\cancel{2}z^{\cancel{2}}}{\cancel{2}\cancel{z}} = z$

Q. 10. (a) (i) $(x + y)(x^2 - xy + y^2) + (x + y)(3)$

$= (x + y)(x^2 - xy + y^2 + 3)$

(ii) $(x - y)(x + y) + 5(x + y)$

$= x + y\,[x - y + 5]$

(iii) $(x - y)^2 - (2z)^2$

$= (x - y - 2z)(x - y + 2z)$

(iv) $(x - y)(x^2 + xy + y^2) + (x - y)(x + y)$

$= x - y(x^2 + xy + y^2 + x + y)$

(v) $(a - b - c)(a + b + c)$

(b) $t_4 = \binom{12}{3}\left(\frac{x}{\sqrt{y}}\right)^9\left(\frac{\sqrt{y}}{2x}\right)^3 = 220\left(\frac{x^9}{y^4\sqrt{y}}\right)\left(\frac{y\sqrt{y}}{8x^3}\right) = \frac{55x^6}{2y^3}$

$\frac{55x^6}{2y^3} > 1 \Rightarrow \frac{55}{2y^3} > 1 \Rightarrow 55 > 2y^3$

$\Rightarrow 27.5 > y^3 \Rightarrow y < 3.018$

The greatest value for $y \in N$ is 3.

(c) $\frac{x^2}{(x - y)(x - z)} + \frac{y^2}{-1(x - y)(y - z)} + \frac{yz}{(-1)(x - z)(-1)(y - z)}$

$= \frac{x^2(y - z) + y^2(-1)(x - z) + yz(x - y)(-1)(-1)}{(x - y)(x - z)(y - z)}$

$= \frac{x^2y - x^2z - y^2x + \cancel{y^2z} + xyz - \cancel{y^2z}}{(x - y)(x - z)(y - z)}$

$= \frac{x^2y - x^2z - xy^2 + xyz}{(x - y)(x - z)(y - z)}$

$= \frac{x(xy - xz - y^2 + yz)}{(x - y)(x - z)(y - z)}$

$= \frac{x[x(y - z) - y(y - z)]}{(x - y)(x - z)(y - z)}$

$= \frac{x\cancel{(x - y)}\cancel{(y - z)}}{\cancel{(x - y)}(x - z)\cancel{(y - z)}}$

$= \frac{x}{(x - z)}$

LHS $x = 1$, $y = 2$, $z = 3$

$\frac{x^2}{(x - y)(x - z)} + \frac{y^2}{(y - z)(y - x)} + \frac{yz}{(z - x)(z - y)}$

$\frac{(1)^2}{(1 - 2)(1 - 3)} + \frac{(2)^2}{(2 - 3)(2 - 1)}$

$\frac{1}{(-1)(-2)} + \frac{4}{(-1)(1)} + \frac{(2)(3)}{(3 - 1)(3 - 2)}$

$= \frac{1}{2} - 4 + \frac{6}{2} = \frac{1}{2} - 4 + 3 = -\frac{1}{2}$

RHS

$\frac{x}{(x - z)} = \frac{1}{(1 - 3)} = \frac{-1}{2}$

LHS = RHS

Verified